Vector

Law of Parallelogram of vectors:

"If a particle simultaneously possesses two vectors represented in magnitudes and direction by the two adjacent sides of a parallelogram drawn from a point, their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point."

Resultant:

Let, $\vec{P}$ and $\vec{Q}$ represented in magnitude and direction along OA and OC through common origin O with angle α, R is the resultant acting along OB with an angle with P From B. The perpendicular BN is drawn on Produce OA. Therefore, form the right angled triangle OBN,

We get,

$$OB^2 = ON^2 + BN^2$$

$$\text{Or, } OB^2 = (OA+AN)^2 + BN^2$$

$$\text{Or, } OB^2 = OA^2 + 2OA.AN + AN^2 + BN^2$$

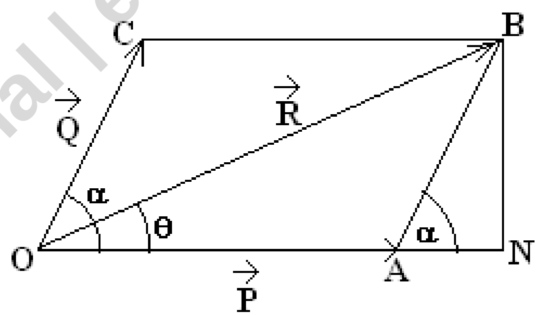
$$\text{Or, } OB^2 = OA^2 + (AN^2 + BN^2) + 2OA.AN$$

$$\text{Or, } OB^2 = OA^2 + AB^2 + 2OA.AB.\frac{AN}{AB}$$

$$\text{Or, } OB^2 = OA^2 + OC^2 + 2OA.OC\cos\alpha$$

$$\text{Or, } R^2 = P^2 + Q^2 + 2P.Q\cos\alpha$$

$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} \quad \ldots \ldots \ldots (1)$$



Direction Resultant:

If the resultant R makes an angle θ with P *i.e.* $\angle AOB = \theta$ then from the right angle triangle, OBN, we get,

$$\tan\theta = \frac{BN}{ON}$$

$$\Rightarrow \tan\theta = \frac{BN}{OA + AN}$$

$$\Rightarrow \tan\theta = \frac{AB\frac{BN}{AB}}{OA + AB\frac{AN}{AB}}$$

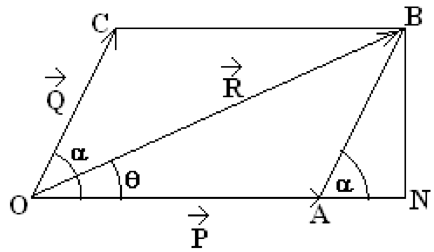
$$\Rightarrow \tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

$$\therefore \theta = \tan^{-1}\frac{Q\sin\alpha}{P + Q\cos\alpha} \quad \ldots \ldots \ldots (2)$$

No. (1) equation is resultant of the vector and No. (2) equation is direction of the resultant.

The maximum and minimum magnitude of the resultant of two vectors acting at a point is equal to the sum and difference of their magnitude Or The resultant of two vectors acting at a point at same time can never be greater than to their sum and can never smaller than to their subtraction.

Let the two vector are P and Q and the angle between them is α. Therefore the law of parallelogram, we know,



$$\therefore R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha} \quad \dots \quad \dots \quad \dots \quad (1)$$

$$\Rightarrow R^2 = P^2 + Q^2 + 2PQ\cos\alpha$$

$$\Rightarrow R^2 - P^2 - Q^2 = 2PQ\cos\alpha$$

$$\therefore \cos\alpha = \frac{R^2 - P^2 - Q^2}{2PQ}$$

$$\text{We Know, } 1 \ge \cos\alpha \ge -1$$

$$\Rightarrow 1 \ge \frac{R^2 - P^2 - Q^2}{2PQ} \ge -1 \quad [\text{By putting the value of } \cos\alpha]$$

$$\Rightarrow 2PQ \ge R^2 - P^2 - Q^2 \ge -2PQ$$

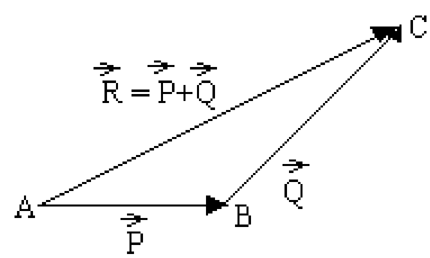
$$\Rightarrow P^2 + Q^2 + 2PQ \ge R^2 \ge P^2 + Q^2 - 2PQ \quad [P^2 + Q^2 \text{ add both sides}]$$

$$\Rightarrow (P + Q)^2 \ge R^2 \ge (P \sim Q)^2$$

$$\therefore (P + Q) \ge R \ge (P \sim Q)$$

So, that the maximum and minimum magnitude of the resultant of two vectors acting at a point is equal to the sum and difference of their magnitude, Or The resultant of two vectors acting at a point at same time can never be greater than to their sum and can never smaller than to their subtraction.

Law of Triangle of vector:



"If a body simultaneously possesses three vectors represented in magnitudes and directions by the sides of a triangle taken in order, the resultant is zero i.e. the body remains at rest. "Let P Q and R be represented by the three sides of triangle AB , BC and CA respectively.

According to vector addition,

$$\vec{AB} + \vec{BC} = \vec{AC} \Rightarrow \vec{P} + \vec{Q} = -\vec{R}$$

$$\Rightarrow \vec{P} + \vec{Q} + \vec{R} = -\vec{R} + \vec{R}$$

$\therefore \vec{P} + \vec{Q} + \vec{R} = 0$ Proved. (By adding R in both the sides)

Proof of $\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Let, $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$ and, $\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

So, $\vec{A}.\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k}).(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$$\vec{A}.\vec{B} = A_xB_x(\hat{i}.\hat{i}) + A_xB_y(\hat{i}.\hat{j}) + A_xB_z(\hat{i}.\hat{k})$$
$$+ A_yB_x(\hat{j}.\hat{i}) + A_yB_y(\hat{j}.\hat{j}) + A_yB_z(\hat{j}.\hat{k})$$
$$+ A_zB_x(\hat{k}.\hat{i}) + A_zB_y(\hat{k}.\hat{j}) + A_zB_z(\hat{k}.\hat{k})$$

$$\Rightarrow \vec{A}.\vec{B} = A_xB_x(1) + A_xB_y(0) + A_xB_z(0) + A_yB_x(0) + A_yB_y(1) + A_yB_z(0) + A_zB_x(0) + A_zB_y(0) + A_zB_z(1)$$

$$\Rightarrow \vec{A}.\vec{B} = A_xB_x + 0 + 0 + 0 + A_yB_y + 0 + 0 + 0 + A_zB_z$$

$$\vec{A}.\vec{B} = A_xB_x + A_yB_y + A_zB_z \quad \text{(Proved)}$$